Experimental studies of occupation times in turbulent flows

J. Mann and S. Ott
Risø National Laboratory, DK-4000 Roskilde, Denmark

H. L. Pécseli
Institute of Physics, University of Oslo, Box 1048 Blindern, N-0316 Oslo, Norway

J. Trulsen
Institute of Theoretical Astrophysics, University of Oslo, Box 1029 Blindern, N-0315 Oslo, Norway
(Received 4 October 2002; published 19 May 2003)

The motion of passively convected particles in turbulent flows is studied experimentally in approximately homogeneous and isotropic turbulent flows, generated in water by two moving grids. The simultaneous trajectories of many small passively convected, neutrally buoyant, polystyrene particles are followed in time by a particle tracking technique. We estimate the probability distribution of the occupation times of such particles in spherical volumes with a given radius. A self-consistently moving particle defines the center of the reference sphere, with the occupation time being defined as the difference between entrance and exit times of surrounding particles convected through the sphere by the turbulent motions. Simple, and seemingly universal, scaling laws are obtained for the probability density of the occupation times in terms of the basic properties for the turbulent flow and the geometry. In the present formulation, the results of the analysis are relevant for understanding details in the feeding rate of micro-organisms in turbulent waters, for instance.

DOI: 10.1103/PhysRevE.67.056307 PACS number(s): 47.27.Gs, 47.27.Qb, 05.45.−a

The problem of turbulent transport of passively convected particles in neutral, turbulent flows can be formulated in different ways, depending on the actual problem. Particular attention has been given to the analysis of Richardson's law for relative diffusion, and more generally to the time evolution of the probability density for separation distances of two initially close particles [1–3]. In the present paper, we consider a different problem which is, however, also related to particle separations, by analyzing the time spent by a particle in a reference volume, which is centered either at a fixed spatial position (the Eulerian case) or at the position of another particle (the Lagrangian case). In the latter case it is assumed that *both* particles passively follow the random turbulent motion of the flow. In lack of any established terminology, we denote this time the "occupation time" in the following. The reference volume can, in principle, have any shape, but we consider here the simplest spherical case, considering different radii. The particular formulation of the relative diffusion problem chosen here has direct relevance for the understanding of the feeding process of aquatic micro-organisms, as discussed later in the present paper.

The basic features of the present experiment are described elsewhere [2], so a summary will suffice here. The tank has $320\times 320\times 450$ mm^3 inner dimensions, and the turbulence is generated by the motion of two plastic grids in the top and bottom of the tank. Typical Taylor microscale Reynolds numbers [4], $R_\lambda=\lambda^2/(\eta^2\sqrt{15})$, are ~ 100 for the present conditions, using the Taylor microscale $\lambda=\sqrt{15\nu\sigma^2/\epsilon}$, where $\nu\simeq 0.89$ mm^2/s is the kinematic viscosity of the water, ϵ is specific energy dissipation, while σ^2 is the variance of one velocity component. The Kolmogorov length scale $\eta=(\nu^3/\epsilon)^{1/4}$ is less than 1/2 mm for the present conditions. A characteristic Eulerian length scale ("outer" scale) $\mathcal{L}_E$ is in the range 20–25 mm. We can interpret $\mathcal{L}_E$ as the lower limit for separations between fixed frame detection points, where velocities of fluid elements tend to become uncorrelated. As a working hypothesis we can assume that these velocities are also statistically independent.

The motions of small diameter ($d=0.5$–0.6 mm) polystyrene particles in the flow are followed with four video cameras. The simultaneous positions of typically 500–1000 particles recorded at time intervals of 1/25 s. By a tracking procedure it is then possible to link the positions of particles, and follow their individual motions in three spatial dimensions, in particular, also to deduce their time varying velocity. The particles used in the experiment are neutrally buoyant. The average distance between particles is much larger than their diameter. To the given accuracy, we can assume that the particles follow the flow as passive tracers [5,6]. The present experiment provides a database, which can be used for studying different aspects of turbulent transport [2,7].

We follow a selected particle in the flow, and define a surrounding self-consistently comoving sphere with prescribed radius $\mathcal{R}$. Determining the entrance and exit times of other particles with respect to the given sphere, we determine the distribution of the times the particles spent inside the sphere. The numerical analysis is lengthy and involves a detailed book-keeping of all particles at all times, but the basic principles are self-explanatory, and need not be discussed in detail here. Results from such an analysis are shown in Fig. 1, with abscissa in seconds. (These results should not be confused with the probability densities for finding a particle inside a selected sphere with radius $\mathcal{R}$). Two cases are chosen so that they cover $\mathcal{R}<\mathcal{L}_E$ and $\mathcal{R}>\mathcal{L}_E$. Due to the sampling, we will not record particles staying less than 1/25 s in the sphere, i.e., "glancing" trajectories. For this reason, the first data point gives an underestimate. With a continuous time resolution, one would find a finite probability for arbi-

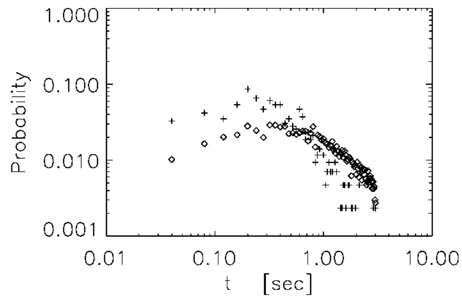

FIG. 1. Experimentally obtained probability densities for the occupation times of particles in a sphere which is moving self-consistently with the flow, i.e., at all times it has the same reference particle in its center. Two cases are illustrated, one where the radius is $\mathcal{R}=10$ mm (+), and one with $\mathcal{R}=30$ mm ($\diamond$). We have $\epsilon=225$ mm^2 s^{-3} and $\sigma=19$ mm s^{-1}. We used the most "honest" normalization by having the sum of all points be unity. If a continuous curve is fitted to the data points, the values should be multiplied by 25 (originating from the 1/25-s sampling rate) to give the normalization of the probability density, $P_L(t)$.

trarily small occupation times. Figure 1 illustrates the scatter in data points for different times and $\mathcal{R}$ values, and serves also to introduce the physical time scales (in seconds), while following figures will present results for scaled variables.

In the universal subrange of the turbulence, where the effect of viscosity is immaterial (in the present case, length scales typically in the interval 0.5–25 mm) we expect that a universal scaling law should exist. We need a universal "time" for normalization, and with the parameters ϵ and $\mathcal{R}$ being the only dimensional parameters available, the only characteristic time available is $\mathcal{R}^{2/3}/\epsilon^{1/3}$. In the universal length interval we expect that the probability density for the occupation times can be written as $(\mathcal{R}^{2/3}/\epsilon^{1/3})P_L(t\epsilon^{1/3}/\mathcal{R}^{2/3})$, assuming that ϵ is a deterministic constant, and thereby ignoring intermittency corrections [1,3]. The conjectured scaling for the occupation time probabilities can readily be tested in the experiment, with results shown in Figs. 2 and 3. If we try to include data for radii $\mathcal{R}>\mathcal{L}_E$, we find a clear disagreement with the proposed scaling. We find an excellent scaling with $\mathcal{R}$ for fixed ϵ, see Fig. 2. When we include different experimental conditions, with different ϵ, we find a slight increase in scatter, which

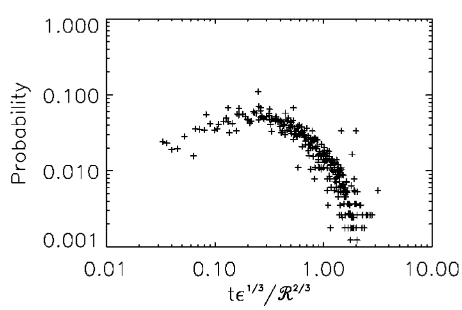

FIG. 2. Experimentally obtained, normalized probability densities for the occupation times of particles in a self-consistently moving sphere, showing $(\mathcal{R}^{2/3}/\epsilon^{1/3})\,P_L(t\epsilon^{1/3}/\mathcal{R}^{2/3})$. The figure refers to experimental conditions with $\epsilon=225$ mm^2 s^{-3}, analyzed for radii $\mathcal{R}=2.5$, 5, 7.5, 10, 12.5, 15, 17.5, and 20 mm.

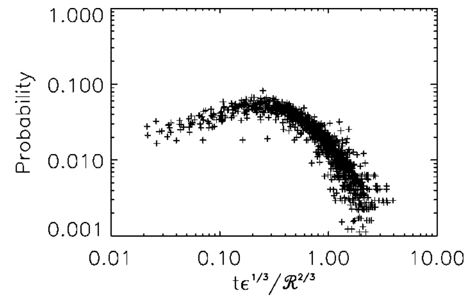

FIG. 3. Experimentally obtained, normalized probability densities for the occupation times of particles in a self-consistently moving sphere, showing $(\mathcal{R}^{2/3}/\epsilon^{1/3})\,P_L(t\epsilon^{1/3}/\mathcal{R}^{2/3})$. The figure contains six experimental conditions with $\epsilon=62$, 65, 135, 160, 225, and 279 mm^2 s^{-3}, each of these analyzed for radii $\mathcal{R}=5$, 10, 15, and 20 mm.

might be an indication of the uncertainties associated with an experimental determination of the dissipation rate, ϵ. We have discussed spherical volumes, in particular, but the scaling arguments apply equally well also for deformed volumes, as far as we are dealing with a self-similar scaling of the entire volume with just *one* length scale, here denoted $\mathcal{R}$.

As already mentioned, the occupation time statistics discussed in the present paper are dealing with only one of many manifestations of relative turbulent transport. There are, however, cases where this formulation is particularly relevant. Our interest in the problem arises in part from discussions of the feeding processes of micro-organisms in turbulent environments. Since, at least in a standard formulation of the problem, both predators and prey are passively convected by the flow in which they are embedded [8], the problem is directly related to relative diffusion of particles, the main difference being that we are here dealing with a *boundary value* problem, where the surface of a suitable defined "sphere of interception" in effect acts as a perfectly absorbing surface. By model studies [9–12], or simple dimensional reasoning [7], it can be argued that that the steady state normalized prey flux to such an absorbing surface must scale as $J/\eta_0\approx C\epsilon^{1/3}\mathcal{R}^{7/3}$, where a numerical constant can be estimated as $C=0.32\pm0.05$ by use of Richardson's model diffusion equation for relative diffusion [2]. The density of prey at infinite distance from the predator is introduced as η_0. (It might be interesting to mention that implicit in the foregoing arguments is that a nontrivial steady state actually exists: this is indeed correct in three spatial dimensions, but not so in two [13].)

It is evident that a nontrivial assumption is implied in these arguments, namely, that preys are captured with certainty within the sphere of interception. Even in the case where the prey concentration is so low that, on average, only one is present at a time (this is the most relevant case), there will evidently be cases where the turbulent transport of prey past the predator is so rapid that a capture is unlikely [14]. For a given predator (with a given range $\mathcal{R}$), this process can be modeled by assuming that given an occupation time t, the probability of capture is $P(\mathrm{capt}|t)=t/\tau_0$ for $t\leqslant\tau_0$ and $P(\mathrm{capt}|t)=1$ for $t>\tau_0$, where τ_0 is a constant time-scale characteristic of the species. Evidently, τ_0 will be different for different species, and therefore also for different $\mathcal{R}$. The probability of capture of prey is then obtained by $P(\mathrm{capt}) = \int P(\mathrm{capt}|t)P(t)dt$ by Bayes theorem, where $P(t)$ is the probability density of occupation times that we obtained experimentally before. With reference to the $\epsilon^{1/3}/\mathcal{R}^{2/3}$ time scaling of the saturated prey flux, there are now two obvious limiting cases: $\tau_0 \ll \mathcal{R}^{2/3}/\epsilon^{1/3}$ and $\tau_0 \gg \mathcal{R}^{2/3}/\epsilon^{1/3}$. In the former case we have $P(\mathrm{capt}) \approx 1$, and the results for J/η_0 apply [7]. In the latter limit we find $P(\mathrm{capt}) \sim \mathcal{R}^{2/3}/(\epsilon^{1/3}\tau_0)$. In this case we can obtain the scaling of the flux J_c actually captured by the predator under steady state conditions as the product of flux and capture probability,

$$J_c/\eta_0 \sim \epsilon^{1/3}\mathcal{R}^{7/3}\frac{\mathcal{R}^{2/3}}{\epsilon^{1/3}\tau_0} \sim \frac{\mathcal{R}^3}{\tau_0}, \qquad (1)$$

i.e., J_c is independent of ϵ. This means that if we start with a predator in calm waters with a corresponding vanishing prey flux, and then slowly increase ϵ by "external" stirring while $\mathcal{R}$ is constant, we first find $J_c/\eta_0 \sim \epsilon^{1/3}\mathcal{R}^{7/3}$ until $\epsilon \approx \mathcal{R}^2/\tau_0^3$. From then on we have J_c/η_0 independent of ϵ. In this limit, an increase in the prey flux is exactly canceled by a corresponding reduction in the capture probability, due to the rapid sweeping of prey past the predator.

We can elaborate Eq. (1) a little further by noting that in order for the turbulent relative dispersion to be of importance for a predator with range of interception $\mathcal{R}$, we require that this length is larger than the Kolmogorov length scale, i.e., $\mathcal{R} \geq \nu^{3/4}/\epsilon^{1/4}$. For the limit given by Eq. (1) we require, as mentioned, $\tau_0 > \mathcal{R}^{2/3}/\epsilon^{1/3}$. This implies $\nu^{3/4}/\epsilon^{1/4} \leq \mathcal{R} < \tau_0^{3/2}\epsilon^{1/2}$, or $\tau_0 > (\nu/\epsilon)^{1/2}$. This means that τ_0 has to be larger than the Kolmogorov time scale $(\nu/\epsilon)^{1/2}$, for the limit (1) to be relevant. This inequality should be amenable for experimental investigations. It should be noted that the Kolmogorov time scale is rather small for most relevant turbulent flows: for the present experimental conditions it is less than 0.1 s. In nature, we might thus experience that the prey flux to a predating micro-organism scales as $\mathcal{R}^3$, independent of the turbulence level, at least in an observable subrange.

We can generalize the previous simple model for the capture probability by allowing for a subrange $(t/\tau_1)^\alpha$ for very small times. With $\alpha \geq 2$ this model accounts for a vanishing capture probability for prey with large relative velocities [15]. It is readily demonstrated, by taking $\alpha = 2$, for instance, that Eq. (1) is modified to

$$J_c/\eta_0 \sim \epsilon^{1/3}\mathcal{R}^{7/3}\left(\frac{\mathcal{R}^{2/3}}{\epsilon^{1/3}\tau_1}\right)^{\alpha=2} \sim \frac{\mathcal{R}^{11/3}}{\epsilon^{1/3}\tau_1^2}. \qquad (2)$$

Within this modified model, we evidently find $J_c \to 0$ for $\epsilon \to \infty$, but an intermediate "plateau" with constant J_c might be anticipated, in general. The result given by Eq. (2) is trivially generalized to arbitrary values for $\alpha \neq 2$. While the case with $\alpha = 1$ discussed before is the easiest one argued for, the more general case can be used for modeling. Our arguments are easily generalized for different capture probabilities for different occupation time subranges, each characterized by some time scale and a time exponent.

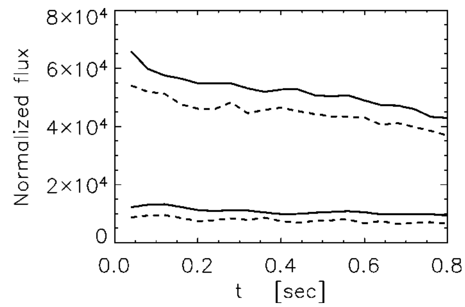

FIG. 4. Experimentally obtained prey flux to a self-consistently moving sphere of interception, assuming that prey has to spend five sampling time units (i.e., 1/5 s altogether) inside the sphere before it is captured, as shown with the dashed line. The full line gives the corresponding result without time delay. Results are shown for two different radii $\mathcal{R} = 15$ and 20 mm in the sphere of interception. The difference between the dashed and full line gives the number of particles that entered at the appropriate time but left the reference sphere within five sampling time units.

The model for the capture probability given before is probably the simplest possible, and supposedly encompassing most realistic models. One objection to the expression (1) or (2) could be that we used the flux-scaling obtained for a *perfectly* absorbing surface in connection with a case where the absorption was only *partial*. It is, however, possible to test also the accuracy of this approximation by performing a flux analysis where it is assumed that prey has to stay for at least a prescribed number of sampling periods before it is "captured." An illustrative result is shown in Fig. 4. The analysis is carried out without "replacement" in a given sequence of data analysis, i.e., a polystyrene sphere representing prey is counted only first time it enters the reference sphere. We see that the reduction in flux is moderate, and well described by a constant fraction. The point is that prey that has not been captured has the possibility of returning at a later time, where it has a new possibility for capture. The reduction in prey flux is therefore not as large as might have been anticipated. *Qualitatively*, the result (1) or (2) therefore remains correct as far as the scaling is concerned, leaving a numerical factor undetermined.

In the present formulation of the problem of relative motion of two particles in turbulent flows, we considered the problem which, in a sense, is *opposite* to the one of particle separation, by investigating the distributions of times that particles spend close together in a turbulent environment. We demonstrated that these distributions contain information that is central for the understanding of, for instance, details of the feeding process of aquatic micro-organisms.

This work was in part supported by the Danish Technical Research Council, under Contract Nos. STVF-9601244 and 26-01-0087. Two of the authors (H.L.P. and J.T.) were in part supported by the "Effects of North Atlantic Climate Variability on the Barents Sea Ecosystem" (ECOBE) project. Valuable discussions with Maria Pécseli are gratefully acknowledged.

[1] G. Boffetta, A. Celani, A. Crisanti, and A. Vulpiani, Phys. Rev. E **60**, 6734 (1999).
[2] S. Ott and J. Mann, J. Fluid Mech. **422**, 207 (2000).
[3] G. Boffetta and I.M. Sokolov, Phys. Rev. Lett. **88**, 094501 (2002).
[4] J.O. Hinze, *Turbulence*, 2nd ed. (McGraw-Hill, New York, 1975).
[5] P.S. Hill, A.R.M. Nowell, and P.A. Jumars, J. Mar. Res. **50**, 643 (1992).
[6] M.R. Maxey and J.J. Riley, Phys. Fluids **26**, 883 (1983).
[7] J. Mann, S. Ott, H.L. Pécseli, and J. Trulsen, Phys. Rev. E **65**, 026304 (2002).
[8] J.H. Muelbert, M.R. Lewis, and D.E. Kelley, J. Plankton Res. **16**, 927 (1994).
[9] B.J. Rothschild and T.R. Osborn, J. Plankton Res. **10**, 465 (1988).
[10] S. Sundby and P. Fossum, J. Plankton Res. **12**, 1153 (1990).
[11] T. Kiørboe and E. Saiz, Mar. Ecol.: Prog. Ser. **122**, 135 (1995).
[12] T. Osborn, J. Plankton Res. **18**, 185 (1996).
[13] The result for the saturated prey flux to a predator [7] refers to three spatial dimensions. If we assume that prey is confined to a layer of thickness $\mathcal{L}$ with $\mathcal{L} \ll \mathcal{R}$, we ignore one coordinate and analyze the problem in two spatial dimensions by a standard diffusion equation with constant diffusion coefficient D, to obtain the turbulent flux through a "cylinder segment of interception" as

$$J(t) = \eta_0 \mathcal{L}\mathcal{R} D \frac{8}{\pi} \int_0^\infty \frac{e^{-\lambda^2 D t}}{J_0^2(\lambda\mathcal{R}) + Y_0^2(\lambda\mathcal{R})} \frac{d\lambda}{\lambda},$$

in terms of Bessel functions J_0 and Y_0. Evidently, the prey flux does not reach a stationary level for $t \to \infty$ in this case, but is steadily decreasing. We expect this observation to be relevant also for models of turbulent diffusion in two spatial dimensions.
[14] T.C. Granata and T.D. Dickey, Prog. Oceanogr. **26**, 243 (1991); T. Kiørboe, Sci. Mar. **61**, 141 (1997).
[15] B.R. Mackenzie and T. Kiørboe, Limnol. Oceanogr. **45**, 1 (2000).